Mathematical Knowledge in Teaching about Fractions

Manchester University, 18th April 2007

An, S., Kulm, G. & Wu, J. (2004) *The Pedagogical Content Knowledge of Middle School Mathematics Teachers in China and the U.S.,* Journal of Mathematics Teacher Education 7(2) 145-172.

In preparation for the seminar, I read the paper by An et al (2004) and considered the study design and conclusions as well as thinking about the content of the paper from my own research perspective. I have included in this summary the main points I presented as an introduction to the seminar discussion on the paper, as well as some of the issues raised during the discussion.

Study Design

The methods used by the researchers were essentially three types: questionnaires, interviews and observations. The questionnaires were based on teachers' approaches to four problems (p.152) and each problem set out a context which required calculations involving fractions. In addition there were some supplementary questions which required each teacher to discuss how they might teach the mathematical knowledge needed to facilitate the calculations and some questions which considered the teachers' views about students' thinking and ways of dealing with misconceptions. Suggestions were included that might lead a respondent to offer a model or range of models they might use to teach the mathematical knowledge.

The interviews were carried out following completion of the questionnaires, and the questions used were designed to elicit information about each teacher's mathematical goals, beliefs about mathematics education and about how they prepared for their teaching sessions.

The observations seemed to act as a triangulation tool in that they were described as being "to confirm that the teachers' responses … were consistent with their actual classroom practices and their knowledge and beliefs about effective teaching" (p.153)

In reading this section of the paper, I was thinking about the compatibility of the data from the methods and whether the observations might have been used to collect a broader range of data beyond simply confirming the data from the questionnaires and interviews, and I offered this as a possible discussion point.

The sample used by the researchers consisted of 28 teachers from grade 5 to 8 in a metropolitan area of Texas and 33 teachers, grade 5 to 6, from a city in Jiangsu province. I was led to consider why the range of grades covered was broader in one set of teachers than in the other, since the curriculum may offer a different range of fraction knowledge and could therefore lead to different views being expressed by the two groups of teachers. There may also be a cultural issue with the comparison of locations providing such diverse cultural norms that a fair comparison may not be possible.

Following the data collection, I was interested in the way the data items were categorised. There were 18 response categories for coding transcriptions, covering a broad variety of responses related to the teachers' responses to the pedagogical content knowledge questions. These 18 categories were then grouped into 4 main components of PCK to be used for analysis. The groupings covered building on students' ideas, addressing misconceptions, engaging students in mathematical learning, and promoting thinking about mathematics. The numbers of categories and the tetra-partheid groupings reminded me of the work on the Knowledge Quartet which Rowland, Huckstep and Thwaites (2002, 2003) produced. I invited Tim Rowland and Peter Huckstep to comment on whether there were similarities in the two studies.

Study Conclusions the researchers considered that they had explored a critical way of assessing teachers' PCK regarding students' thinking through their use of the questionnaires, interviews and observations. In particular, they regarded the PCK questions crucial in determining the extent to which teachers consider PCK issues when planning their lessons.

A major recurrence for me throughout the paper was the consideration of the approaches taken by the teachers in the United States and China, and how the Chinese approach was often referred to as being more effective. Whilst the researchers consider that a balance is needed between the uses of manipulatives and developing procedures, there seems to be a suggestion that the two extremes are mutually exclusive. Further, they suggest that whilst manipulatives develop conceptual understanding, procedural learning is essential for reinforcing understanding and for proficiency, and is a necessary step towards problem solving. The Chinese seem to emphasise proficiency ahead of skills for problem solving.

Another closely related conclusion presented was that PCK was markedly different in the two countries, and this has an impact on teaching practice. Namely, the Chinese emphasize rigid procedural knowledge but the US emphasises creativity and inquiry for concept mastery, but lacks a connection between manipulatives and abstract thinking.

Thoughts from my perspective…

I was interested in comparing the methods used from a teacher educator perspective. I work mostly with students learning to teach maths to Key Stage 1 and 2, and so some aspects of fractions, particularly involving multiplication and division of fractions, rarely has time for coverage in our course. However, the findings of this paper may be given some consideration regarding the amount of fraction calculation that should be included, given the range of pedagogical issues that can be raised.

My own doctoral research is related to earlier work by Rowland et al, Watson and Mason, Zaslavsky, Bills and Bills, and so on, regarding the choice of examples that teachers or trainees might make when presenting material to their students. The question that this paper raised in my mind was 'do the approaches emphasized on ITT programmes enable trainees to have the required conceptual understanding to select appropriate examples?', and I was hoping that the ensuing discussion might offer some broader perspectives on that issue.

Finally in my seminar introduction, I offered some summarised issues for discussion as follows:

Comparison of the Chinese and US approaches, i.e. procedural v. creative/inquiry.

Apparent bias of the authors, implying that the Chinese approach is best!!

Difficulty of fair comparison from the sample and the respective education systems

Comments in relation to any PCK emphasis in the UK.

Summary of discussion points

The first point to be made following the introductions, by Maria Goulding, was that the international comparisons show that Chinese students do perform better than most other students!

Anne Watson then outlined the following points: the paper is a good model of how not to write a paper(!), the misconceptions outlined could perhaps have been more clearly dealt with, perhaps through consideration of how we read and interpret information, and also that we tend to cover too much content at a shallow level compared with the Chinese, and perhaps we should review course content and spend more time focussing on some aspects of PCK in greater depth.

Tim Rowland described how despite the fact there were only 4 problems used in the data collection questionnaire, it in fact produced so much data that fewer problems could have been used to be able to analyse effectively. He also raised the issue of the categories being pre-determined rather than identified as part of the analysis.

This last point was backed up by Jeremy Hodgen who felt that the outcomes of the research were pre-determined and then the research merely mustered the evidence to support the outcomes!

Some clarification was given by Lara Alcock, Ken Ruthven and Tim Rowland of the amount of mathematics a training teacher might encounter in the United States. For example, '24 hours course work' would generally mean 8 x 3 hours per week per semester. This would include calculus, linear algebra and so on for a 'maths major', but along with professional studies courses and a range of support modules, mathematics could take up only around 20% of a maths degree.

Birgit Pepin brought out a consideration of the types of learning which were implied throught the research. 'Learning as knowing' and 'learning as understanding' are offered as alternatives, but Birgit suggested these are not distinct but should be considered as on a continuum.

Cultural differences were then explored for a few minutes, with Johannes Siemons offering the historical perspective of Euclid's geometry not being well received by the Chinese, and that the Chinese had better developed number theory. Maria Goulding reflected that whilst cultural differences are interesting, they are very difficult to change. Julie Ryan talked of 'cultural cringe', and then suggested that all teachers struggle with the 'manipulatives – mental – written' development. She offered the thought that the emphasis is on speed, and that we must slow the pace and allow for thinking time, and also that ITP's are not always good representations for discussing fractions.

Comparison with other European countries then dominated for a while, and Paul Andrews firstly pointed out that in Hungary, 1.75 would be described as '1 and 75 hundredths'. Anne Watson shared that in Scandinavia amongst other areas, 1.75 is called 'one point seventy-five'. There was also the thought shared that the answer to '$0.3 \div 0.8$' could be correctly given as 3/8, but this was unlikely to be given as correct in UK SATS since the question is designed to test manipulation of decimals in particular, rather than number generally.

The final part of the discussion centred on the whole issue of pedagogy and where it should feature in teachers' development. Johannes Siemons offered the view that PCK lacks a mathematical foundation and therefore any basis for discussion. He posed the question the question 'do we learn mathematics and then how to teach it, or do we learn mathematics with pedagogy?' Anne Watson responded by asking whether we can 'pedagogise' mathematics, given that we don't consider in sufficient depth task design, sequences of lessons, etc. Birgit Pepin again raised the issue of time, asking whether it is better to have depth of experience or cover as much as possible? Should we dip into lots of mathematical topics or should we cover just a few in depth to develop critical thinking about the topic? Ros Sutherland agreed that it was impossible to cover everything in depth.

Jeremy Hodgen ended the discussion by raising an interesting point on the theme of confusion, saying that confusion is helpful in the classroom and in mathematics education as it gives thinking time and helps us reflect about how and why we do things.

Ray Huntley

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