Mathematical knowledge in teaching about fractions

Discussion Group theme - developing and deepening mathematical knowledge in teaching

In order to address this theme we focused attention on responses to the question, how can mathematics education develop approaches for addressing the problems identified in the literature?

A number of issues were discussed, drawing on conversations earlier in the day and the papers themselves. We did not attempt to distinguish between primary teachers, who are generally generalists and secondary teachers who are generally specialists in respect of subject knowledge.

The first issue discussed was whether or not teacher education programmes should attempt a broad, but necessarily shallow, coverage of subject knowledge issues or attempt a deeper, but necessarily restricted coverage. Unequivocally the group agreed that depth was essential although what that might mean in practice is not simply defined. Clearly there are areas of content knowledge, as in the division of fractions discussed during the day, that are problematic across, it would seem, Anglophone educational systems. However, Simon offers an interesting perspective on this in describing moments in a child's mathematics experience which involve significant shifts in understanding. Examples of these could be the transition from addition to multiplication or the shift from generalisation to abstraction. Whatever the decisions, in this regard, the group felt that in order to justify the decision to focus on depth a framework within which students or trainees would be able to work critically would be essential. The development of this framework work not be unproblematic and would have to consider, for example, strategies for adapting others' resources for one's own particular classroom needs, ways of analysing critically the myriad resources found on the internet. Included in this process would need to be strategies for predicting the outcomes of activities in order to ensure a coherent sequence and flow of ideas. This is not a simple skill and is closely related to one's understanding of the mathematical ideas underpinning a task or activity.

The second issue, and not unrelated to the first, concerned the desirability of a mathematics education curriculum. The group acknowledged that there was, in essence, a curriculum of sorts embedded in legislation governing teacher education but felt that this was inadequate in many respects and in need of warranted revision. A significant issue that would frame any such curriculum would consider the common characteristics of countries in which the majority of their students were able to demonstrate some measure of mathematical sophistication. While we are not unaware that this is a not unproblematic venture a common characteristic of such systems appeared to lie in the extent to which learners were exposed to coherently structured and intellectually challenging mathematics. Differences in teacher practices were not necessarily indicative of success, although they cannot be dismissed as irrelevant, as countries like China appear to privilege procedures while Japan appears to emphasise concepts. On the one hand, within the explicit procedural emphasis of Chinese lessons lies a subtle but very powerful model of conceptual development related to what are called bianshi problems whereby each successful problem presented to learners embodies a subtle but planned shift of conceptual emphasis. Such a shift is not

unrelated to Marton's variation. On the other hand, within the explicit conceptual emphasis of the Japanese lesson lies a clear intention on the part of the teacher to derive procedures for use with subsequent problems.

Thirdly, there was a consensus that the distinction between subject matter knowledge and pedagogic content knowledge is not always helpful as teachers with a deep and transformative subject knowledge would have, embedded within it, a variety of representations which, essentially, provide a pedagogic tool.

Fourthly, another characteristic of countries which perform well tends to be an underpinning principle that all students should have equal access to an intellectually robust and coherent curriculum. Such expectations, which are frequently realised in countries other than England, run counter not only to the political expectations of successive governments but also most teachers' beliefs about teaching and learning. The group felt it was important to develop strategies to overcome the systematic barriers placed in front of too many learners in this country. However, beliefs are difficult to shift although it was acknowledged that changes in practice are longer lasting when accompanied by shifts in perception.

Overall, though, the dominant theme to emerge from our discussion was the need for teachers to have a developed awareness of the importance, in children's learning, of coherently structured and intellectually challenging mathematics. One of the problems of attempting in-depth analyses of a single issue like division of fractions lies in its being treated in isolation of not only previous work on fractions but also all other previous work. Thus, for example, where key concepts like equivalence are emphasised constantly, problems with individual topics are lessened, particularly if the emphasised concepts permeate as many aspects of the broader topic as possible.

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