Nuffield Seminar Series on Mathematical Knowledge in Teaching
Seminar #6: Formulating a research agenda on mathematical knowledge in teaching

Towards a research programme for identifying what mathematical knowledge is important for teaching[1]

Andreas J. Stylianides, University of Oxford
e-mail: andreas.stylianides@education.ox.ac.uk

I argue that a major limitation of the existing body of research on mathematical knowledge for teaching is the lack of a cohesive methodological/research process for identifying critical elements of mathematical knowledge that can enable teachers to effectively support student learning of mathematics in their classrooms. The outcomes from application of such a cohesive process could inform the curriculum of initial teacher education courses for prospective teachers. There is nowadays significant variation in the elements of mathematical knowledge that are promoted in such courses; this variation is in part due to the absence of a research base that would elevate the importance of certain elements compared to others.

There are several possible approaches to the problem of identifying what mathematical knowledge is important for teaching. Table 1 summarizes seven such approaches that have roughly different objects and methods of inquiry. In order for the overall contribution of these and other possible approaches to be maximized, it is necessary to develop a systematic research programme that would integrate the different approaches and that would embody understanding of the complex network of relationships among them. A plain comparison of Approaches 5 and 7 exemplifies an aspect of this network of relationships. Approach 5 supports inferences about elements of mathematical knowledge that are important for teaching based on what teachers are expected to teach (i.e., the provisions of the school curriculum), whereas Approach 7 supports a direct investigation of this issue by focusing on the actual work that teachers do in their classrooms (i.e., the ways in which teachers implement the provisions of the school curriculum).

How might the different approaches come together in an integrative and systematic programme of research? Subsidiary questions include: How might the research programme resolve discrepancies between the outcomes of different approaches, taking into consideration also the underlying assumptions of these approaches? To what extent and in what ways might the research programme depend on the school level (primary versus secondary), the cultural context, etc.?

I conclude with an important caveat. The potential contributions of the research programme for which I argued herein are constrained by several factors. For example, the identification of some elements of mathematical knowledge that are important for teaching is a matter of value judgments and, therefore, this identification cannot be justified based on research.

[1] This contribution uses ideas discussed in a paper that I co-authored with Deborah Ball and presented at the 2004 Annual Meeting of the American Educational Research Association (San Diego, California).

Table 1
Seven Approaches to the problem of identifying what Mathematical Knowledge is Important for Teaching

Approach #	Object of inquiry	Method of inquiry	Short description
1	Mathematics as a body of knowledge	Mathematical analysis of mathematical topics	Mathematical analysis of topics related to (not necessarily included in) the school curriculum to identify 'key' ideas that teachers might need to understand in order to be able to effectively support student learning of these (or related) topics.
2	Policy documents that set standards for ITE	Text analysis	Text analysis of policy documents to identify the mathematical knowledge that policy makers recommend/require that prospective teachers have an opportunity to develop in ITE. These recommendations/requirements reflect (to some extent) what mathematical knowledge policy makers consider important for teaching.
3	University mathematics curricula (e.g., course syllabi, assessments) used in ITE courses for prospective teachers	Curriculum analysis	Curriculum analysis to identify the opportunities designed in ITE programmes for prospective teachers to develop their mathematical knowledge. These opportunities reflect (to some extent) what mathematical knowledge teacher educators consider important for teaching.
4	Researchers' justifications for the importance of focusing research studies on particular aspects of teachers' mathematical knowledge	Literature review	Literature review of empirical studies on teachers' mathematical knowledge to identify reasons for which researchers explored, or designed interventions to promote, particular aspects of teachers' knowledge. These reasons reflect (to some extent) mathematical knowledge that researchers consider important for teaching.
5	School mathematics curricula (including national frameworks, assessments, student textbooks, and teacher editions/guides)	Curriculum analysis	Curriculum analysis to identify the opportunities designed for students to learn mathematics. Considering what might be involved in the effective implementation of these opportunities in classrooms offers insight into mathematical knowledge that is important for teaching.
6	Research findings about common student conceptions (including misconceptions)	Literature review	Literature review of empirical studies on students' mathematical knowledge to identify common student conceptions about mathematical topics related to the school curriculum. Considering what might be involved in teachers' work in classrooms to foster the development of students' conceptions offers insight into mathematical knowledge that is important for teaching.
7	Successful teaching practices	Mathematical analysis of classroom records	Mathematical analysis of classroom records from successful teaching practices to identify the mathematical demands that are placed on teachers' knowledge as they try to promote student learning of mathematics.